UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Ordinary Level

ADDITIONAL MATHEMATICS **4037/02**

Paper 2 **May/June 2008**

2 hours

Additional Materials: Answer Booklet/Paper
Mathematical tables

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Write your answers on the separate Answer Booklet/Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of
angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

This document consists of **5** printed pages and **3** blank pages.

UNIVERSITY *of* CAMBRIDGE
International Examinations

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Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \ .$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1}\,b + \binom{n}{2}a^{n-2}\,b^2 + \ldots + \binom{n}{r}a^{n-r}\,b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \dfrac{n!}{(n-r)!r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1.$$
$$\sec^2 A = 1 + \tan^2 A.$$
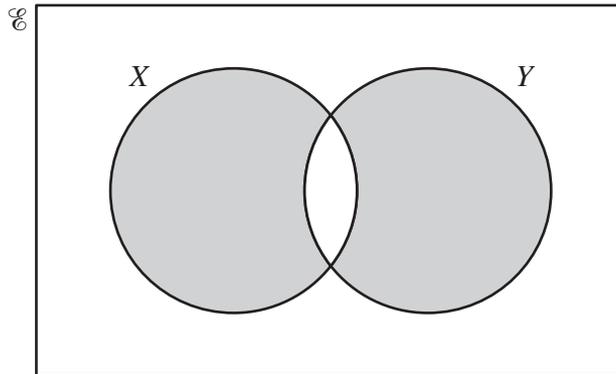$$\operatorname{cosec}^2 A = 1 + \cot^2 A.$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}\,.$$

$$a^2 = b^2 + c^2 - 2bc \cos A.$$

$$\Delta = \frac{1}{2}\,bc \sin A.$$

4037/02/M/J/08

1 The equation of a curve is given by $y = x^2 + ax + 3$, where a is a constant. Given that this equation can also be written as $y = (x + 4)^2 + b$, find

    (i) the value of a and of b, [2]

    (ii) the coordinates of the turning point of the curve. [1]

2 **(a)** Illustrate the following statements using a separate Venn diagram for each.

      (i) $A \cap B = \varnothing$,   **(ii)** $(C \cup D) \subset E$. [2]

    (b)

    Express, in set notation, the set represented by the shaded region. [2]

3 Find the coordinates of the points where the straight line $y = 2x - 3$ intersects the curve $x^2 + y^2 + xy + x = 30$. [5]

4 **(i)** Sketch, on the same diagram, the graphs of $y = x - 3$ and $y = |2x - 9|$. [3]

    (ii) Solve the equation $|2x - 9| = x - 3$. [2]

5 Find the coefficient of x^3 in the expansion of

    (i) $(1 + 3x)^8$, [2]

    (ii) $(1 - 4x)(1 + 3x)^8$. [3]

6 **(a)** Given that $\sin x = p$, find an expression, in terms of p, for $\sec^2 x$. [2]

    (b) Prove that $\sec A \operatorname{cosec} A - \cot A \equiv \tan A$. [4]

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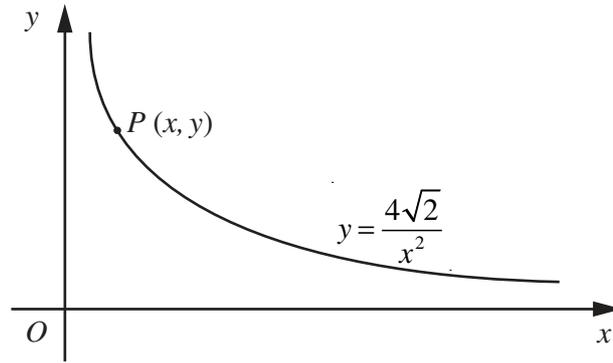
The diagram shows part of the curve $y = \dfrac{4\sqrt{2}}{x^2}$. The point $P\,(x,\,y)$ lies on this curve.

(i) Write down an expression, in terms of x, for $(OP)^2$. [1]

(ii) Denoting $(OP)^2$ by S, find an expression for $\dfrac{\mathrm{d}S}{\mathrm{d}x}$. [2]

(iii) Find the value of x for which S has a stationary value and the corresponding value of OP. [3]

8 Solve the equation

(i) $2^{2x+1} = 20$, [3]

(ii) $\dfrac{5^{4y-1}}{25^y} = \dfrac{125^{y+3}}{25^{2-y}}$. [4]

9 Given that $\mathbf{A} = \begin{pmatrix} 4 & 1 \\ 2 & 3 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 3 & -5 \\ 0 & 2 \end{pmatrix}$ and $\mathbf{C} = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$, calculate

(i) $\mathbf{AB}$, [2]

(ii) $\mathbf{BC}$, [2]

(iii) the matrix $\mathbf{X}$ such that $\mathbf{AX} = \mathbf{B}$.

[4]

10 (a) Find

(i) $\displaystyle\int \dfrac{12}{(2x-1)^4}\,\mathrm{d}x$, [2]

(ii) $\displaystyle\int x(x-1)^2\,\mathrm{d}x$. [3]

(b) (i) Given that $y = 2(x-5)\sqrt{x+4}$, show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{3(x+1)}{\sqrt{x+4}}$. [3]

(ii) Hence find $\displaystyle\int \dfrac{(x+1)}{\sqrt{x+4}}\,\mathrm{d}x$. [2]

11 The function f is defined by

$$f(x) = (x + 1)^2 + 2 \text{ for } x \geqslant -1.$$

Find

(i) the range of f, [1]

(ii) $f^2(1)$, [1]

(iii) an expression for $f^{-1}(x)$. [3]
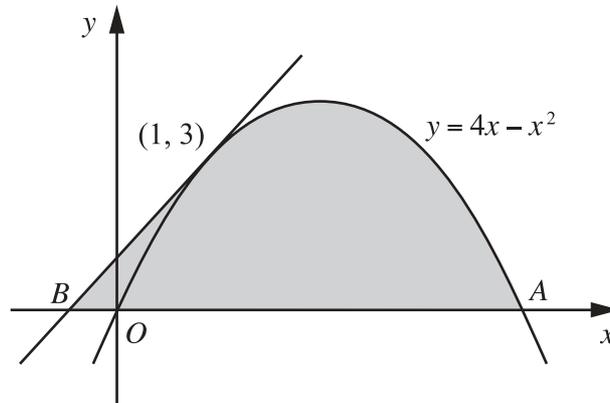
The function g is defined by

$$g(x) = \frac{20}{x + 1} \text{ for } x \geqslant 0.$$

Find

(iv) $g^{-1}(2)$, [2]

(v) the value of x for which $fg(x) = 38$. [4]

12 Answer only **one** of the following two alternatives.

EITHER

The diagram shows the curve $y = 4x - x^2$, which crosses the x-axis at the origin O and the point A. The tangent to the curve at the point $(1, 3)$ crosses the x-axis at the point B.

(i) Find the coordinates of A and of B. [5]

(ii) Find the area of the shaded region. [5]

OR

Solutions to this question by accurate drawing will not be accepted.

The points $A(-2, 2)$, $B(4, 4)$ and $C(5, 2)$ are the vertices of a triangle. The perpendicular bisector of AB and the line through A parallel to BC intersect at the point D. Find the area of the quadrilateral $ABCD$. [10]

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